

# LEARNING MATH THROUGH RESEARCH AND COOPERATION

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**Abstract.** *MATH.en.JEANS/ MeJ (Méthode d'Apprentissage des Théories mathématiques en Jumelant des Etablissements pour une Approche Nouvelle du Savoir) are workshops for students of different age groups. These workshops encourage the students to engage in and eventually learn math by discovering and researching it. The MeJ workshop develops students' creativity, initiative, critical thinking, problem solving skills, etc., and gives students the chance to exchange ideas by working in groups both within their MeJ workshop and with students from a different MeJ workshop. MeJ workshops have a long history in France, where they started in 1985, and a very brief history in Romania. In 2013, we began to plan joint MeJ workshops through a bilateral collaboration between my school and a French high-school, which is continued via an Erasmus+ project. This paper presents our project, our experience in developing MeJ workshops and the results achieved so far.*

*Key words: learn, math, research, collaboration, creativity, workshops.*

## THE PROJECT – IN BRIEF

During the 2013 - 2014 school year, a MeJ workshop was organized in Romania through a bilateral collaboration between Lycée d'Altitude Briançon (France) and Colegiul National Emil Racovita Cluj-Napoca (Romania) with the financial support of the French Institute in Romania, the MeJ Association (<http://www.mathenjeans.fr/>) and the Animath Association (<http://www.animath.fr/>). Lycée d'Altitude had been implementing MeJ activities since 1998 and was looking for a MeJ workshop in Romania to collaborate with. Colegiul National Emil Racovita was interested in developing a math project in collaboration with a European country, for improving the students' transversal and math skills.

The aims of our project were:

- to introduce students to a different way of doing math;
- to show students, by doing math research, that math is wonderful;
- to develop students' curiosity and enjoyment of doing math through a method which gives space to autonomy and imagination;
- to introduce students to academics so they would better understand math research careers.

The main activity of the project was to set up and implement the two collaborative MeJ workshops. The workshops involved:

- two high-schools (Lycée d'Altitude Briançon and Colegiul National Emil Racovita Cluj-Napoca);
- 20 high-school students (volunteers): 10 students (aged 17 – 18) from Colegiul National Emil Racovita and 10 students (aged 16 – 18) from Lycée d'Altitude;
- 2 teachers: Mr. Hubert Proal from Lycée d'Altitude and myself;

- 3 researchers: Mrs. Adela Lupescu from the Babes-Bolyai University, Mr. Camille Petit from the Fribourg University and Mr. Yves Papegay from INRIA – Sophia Antipolis Méditerranée research centre;
- 10 research topics proposed by the French researchers;
- weekly workshop activities in both schools – in each school, the students met with the teacher to discuss the stage of their research; students in groups of two or three carried out the research during and in between the weekly workshops;
- 2 video-conferences which allowed students to share their work with their mates from the other school;
- meetings with the researcher: 12 face-to-face meetings in Cluj-Napoca and 4 video-conferences in Briançon for discussing the status of the research;
- a presentation of the research work and results in each school;
- participation in the Lyon Congress for sharing the experience and the results of the research; the students from both schools prepared and made together presentations on each research topic;
- four research papers written jointly by the students who researched the same topics;
- publication of the research papers.

### The MeJ workshop – principles and practical aspects

The MeJ workshop allows students to meet researchers and experience an authentic math-research process in school, with both a theoretical and an applied dimension. Of major importance in the MeJ workshop are the research topics. They are math-research problems which are formulated differently from the math problems the students are used to. For example, two of the topics the students researched were as follows:

**The vaults.** We have stones of polygonal shapes (except rectangle) and we have to build a vault between two pillars. A stone is in equilibrium if the perpendicular bisectors of the contact surfaces and the vertical line passing through the center of gravity of the stone are concurrent. To figure out how to build such a vault, study its shape. (fig.1)

**Sujet 3 – Les voûtes.** Nous disposons de pierres de formes polygonales (sauf rectangle) et on doit réaliser une voûte entre deux piliers. Une pierre est en équilibre si les médiatrices des surfaces de contacts et la droite verticale passant par le centre de gravité de la pierre sont concourantes.

Comment réaliser de telle voûte, étudier la forme des constructions.

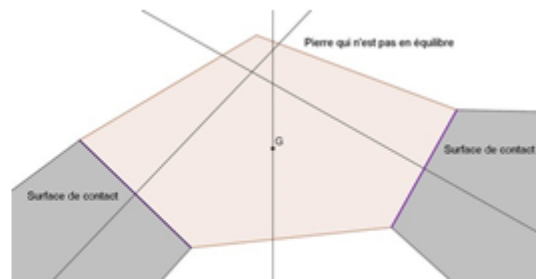
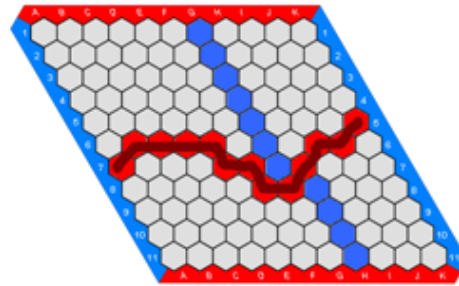


Fig. 1: Research topic 3 - The vaults

**The game of Hex.** Hex game is a board game played on a diamond grid with hexagonal cells. There are two players: a blue one and a red one. Players take turns placing a stone of their colour on a single cell. The goal of the game, for the 'red player', is to form a connected path of red stones linking the opposing sides of the board marked with red and vice versa for the blue player. Develop a winning strategy.

Sujet 7.1 – Le jeu de Hex.

Le jeu de Hex se joue sur un damier en forme de losange dont toutes les cases sont hexagonales. Il y a un joueur bleu et un joueur rouge. Chaque joueur, à tour de rôle, colorie une case du damier avec sa couleur. Le but du jeu, pour le joueur rouge, est d'arriver à relier les deux côtés rouges du damier par un chemin constitué de cases rouges et vice versa pour le joueur bleu.



Mettre en place une stratégie gagnante.

Fig. 2: Research topic 7.1 - The game of Hex

In brief, a MeJ workshop is organized as follows: it lasts throughout the school year; students register voluntarily – there is no student selection; math research topics are launched by professional researchers after being discussed with the teachers; students get a math research topic by choosing what they want to work on; students work in groups for doing the research & collaborate with students from another MeJ workshop who research the same topic; teachers and professional researchers facilitate student research; students share their research results in the annually organized MeJ Congress and different scientific events; students write and publish a scientific article about their research findings.

The MeJ Congress is an important part of the MeJ workshop. One of the reasons is very well pointed out by Mr. Hubert Proal in his report on the 2013 – 2014 MeJ workshop:

The conference is a unique moment of synthesis and evaluation which puts across all the research efforts since the beginning of the year. Its preparation gives an opportunity to each student to raise awareness of the achieved results (by himself, by his group or the twin group) and to reorganise the new knowledge related to both the content and the process of the research. To present the research work and results, students need to focus on the main results, order the stages of the research process, and find an appropriate way to reach the audience. Moreover, the students have to be able to receive the critical feedback of a wider scientific community.

A MeJ workshop is a replica of math research done by professional researchers. This is why the last stage of the MeJ workshop also includes a learning activity in which writing plays a vital role. The students write their articles, which are sent to be validated by the researcher – this is when the researcher gets a comprehensive image of the students' work – and then submitted to an editorial board. The students' articles are published in the MeJ publication or in the Quadrature magazine. You can find last year's articles at <http://www.lyc-altitude.ac-aix-marseille.fr/spip/spip.php?article91>.

Cet article est rédigé par des élèves. Il peut comporter des oublis et imperfections, autant que possible signalés par nos relecteurs dans les notes d'édition.

**Le jeu de HEX  
2013-2014**

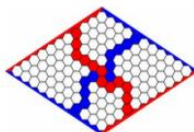
Par Roxana NEMES, Aiana MELNIC et Ionuț COVACI de classe 11 (équivalent terminale) du Colegiul National Emil Racovita de Cluj (Roumanie) et Julia CLAPASSON, Margot ISSERTINE, Thais MATTANA, Tom FERARI et Justin FINE, élèves de seconde du lycée d'Altitude de Briançon (France)

Établissements : Colegiul Național Emil Racoviță (Cluj / România) avec Lycée d'Altitude de Briançon (France)

Enseignants : Ariana Văcărețu (Roumanie) et Mickaël Lissonde et Hubert Proal (France)  
Chercheurs : Madame Adela Lupescu (Université Bades – Bolyai de Cluj-Napoca), Messieurs Camille PETIT (Université de Fribourg) et Yves PAPEGAY (INRIA-Sophia Antipolis)

**Présentation du sujet**

Le jeu de Hex se joue sur un damier en forme de losange dont toutes les cases sont hexagonales. Il y a un joueur bleu et un joueur rouge. Chaque joueur, à tour de rôle, colore une case du damier avec sa couleur. Le but du jeu, pour le joueur rouge, est d'arriver à relier les deux côtés rouges du damier par un chemin constitué de cases rouges et vice versa pour le joueur bleu. Mettre en place une stratégie gagnante.



**Résultats obtenus**

Pour commencer nous avons essayé de trouver une stratégie gagnante en comptant le nombre de combinaisons possibles. Par la suite nous avons développé une stratégie pour que le joueur qui commence soit sûr de gagner (plateau impair x impair), mais cette stratégie n'est pas facile à expliquer.

**Valorisations des travaux**

Les deux groupes ont présenté leurs travaux lors de la semaine des maths. Ils ont animé conjointement leur stand lors du congrès MeJ de Lyon 2014 où ils ont aussi fait un exposé commun.



Les groupes de Cluj et Briançon lors de leur présentation au congrès de Lyon

**Texte de l'article**

**I. Décompte du nombre de combinaisons**

La première démarche pour essayer de trouver une stratégie gagnante a été de lister toutes les combinaisons possibles. Il va de soit que nous avons considérablement réduit la forme du plateau. Par exemple pour un plateau de 3x3, nous avons alors 9 cases, au maximum les rouges (qui commencent) posent 5 points et les bleus 4. Nous essayons de montrer que dans ce cas il y a toujours un joueur gagnant. Nous avons essayé de réaliser un arbre où chaque « colonne » correspond au numéro de la case et ensuite B pour bleu et R pour rouge (figures 1, 2 et 3).

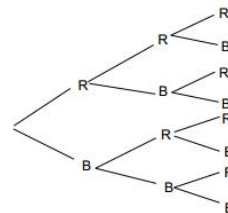


Figure 1 : début de la réalisation de notre arbre

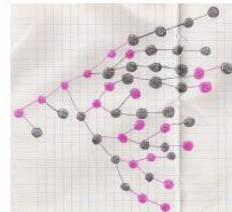


Figure 2 : copie du cahier de recherches

Malheureusement la construction de l'arbre s'est avéré assez compliqué. Nous sommes arrivés à trouver pratiquement 85 positions possibles, il reste à savoir si toutes sont gagnantes.

Fig. 3: The first two pages of the article “The game of Hex”

**The weekly workshop activity in the school**

When I launched the project, my main concern was how to organize and develop the weekly workshop in the school, as it was different from math lessons. It was crystal clear to me what the students had to do: work in groups; read the research topic; translate it into Romanian; discuss it; check if they have the same understanding; translate it into mathematical language; and divide the tasks among the group members. (fig. 4)

I learned from my French colleague that, as the teacher, I would always have to be there, that I should not translate the problem, either into Romanian or into math language, that I should not transform the research topic into a sequence of questions.

During these activities, I created an environment which was appropriate for research and discussions without giving answers to the students' questions. I encouraged the students to communicate their thoughts, to rephrase their colleagues' statements or questions, to allow enough time for their colleagues to talk. I supported the students to formulate questions, to find appropriate ways to model the problem to communicate the results. Thus, I found out that the teacher encourages, comforts, invites to discussions, invites to rigorous proof, (sometimes) suggests the tools if the students ask for it, advises in organizational matters and in the presentation of the results.

It was important for me to keep the students involved in the research, to insist on accuracy and on regular attendance of all students. Students registered in the research project voluntarily, but once they did, they had to do the work.



Fig. 4: Weekly workshop activity



Fig. 5: Researcher during a weekly activity

The researcher participated in almost all the weekly workshop activities (fig. 5). It wasn't necessary to do it, but she was very enthusiastic about the students' work and wanted to be with us in all the project activities. She also encouraged students to formulate questions related to the research topic, provided feedback on their work in progress, and validated, together with the French researchers, the findings of the students' research and their research articles.

## Conclusions

During the one-year bilateral collaboration between Colegiul National Emil Racovita and Lycée d'Altitude, we identified the following issues:

- low and average achievers in math were very interested in participating in the MeJ workshop;
- students' motivation to learn math increased when researching math topics within the MeJ workshop; some students that were low achievers in math showed increased motivation for improving their math academic performance while/ after being part of the MeJ workshop;
- we could not provide clear evidence for the transversal skills and math competences the students developed in the MeJ workshop as we had no system (methods & tools) for assessing the development of the students' skills through the MeJ workshop;
- collaboration between students from two MeJ workshops supports students' research process and math learning; if the MeJ workshops run in parallel in different countries, the students' collaboration yields additional value, as they learn about a different culture and communicate on everyday topics and on math-related topics in a foreign language.

The students' MeJ workshop experience is well illustrated by one student's reflection:

When we joined the project in the 11<sup>th</sup> grade, we didn't have a single idea about what a mathematical project was or about how we should do our research. During the months in which we completed our project, we learned that sometimes the ideas that seem the greatest are the ones that fail first. We worked hard to get some relevant results before the Lyon Congress. At the conference, we were pleasantly surprised. The conference was filled with students, researchers and teachers from all over France, and we also had the opportunity to meet with Romanian researchers who were working there. At first, we had problems communicating with the students from France, but we quickly got over the language barrier. All in all, the whole project was for us a great experience, and we are proud to be among the first 10 Romanian students to have participated in this project.

In September 2014, we continued the implementation of the MeJ workshops as part of the Erasmus+ project Learning math and languages through research and cooperation – MatLan, funded with support from the European Commission. The MatLan project (2014-1-RO01-KA201-002699) builds on the experience gained in the MeJ workshops aiming to valorize students' creativity and innovation by inviting them to discover and research mathematics. Mathematics in the classroom is too abstract for the students to feel attracted to, but in the workshops they get to mathematically investigate issues that no-one has an answer to yet. MatLan adds a series of new dimensions to the MeJ experience: **the assessment dimension**: we will develop guidelines for assessing students' skills (transversal skills and mathematical competences) developed through mathematical research within the MeJ workshops; **the inter-cultural dimension**: we will encourage the inter-cultural exchanges around mathematical issues; **the multilingualism dimension**: we will create opportunities for the students' language learning through collaboration within mathematical research activities; **the formal education dimension**: we will prove and promote a possibility for including the MeJ workshop (non-formal education) into the school curriculum (formal education).

In September 2014, we had a very good start with the MeJ workshop at Colegiul National Emil Racovita as 78 high-school students (aged 15 – 18) enrolled in the MeJ workshop. The good start gives us hopes for the successful implementation of the MatLan project.

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