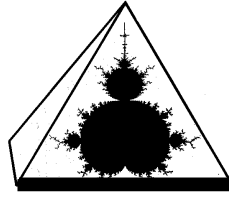


The Mathematics Education for the Future Project



Proceedings of the 13th International Conference

Mathematics Education in a Connected World

Sep 16–21, 2015, Grand Hotel Baia Verde, Catania, Sicily, Italy

Edited by Alan Rogerson

The Mathematics Education for the Future Project thanks our Major Sponsor

Autograph



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Foreword

This volume contains the papers presented at the International Conference on *Mathematics Education in a Connected World* held from September 16-21, 2015. The Conference was organized by The Mathematics Education for the Future Project - an international educational project founded in 1986. Our Project is dedicated to the improvement of mathematics education world-wide through the publication and dissemination of innovative ideas. Many prominent mathematics educators have supported and contributed to the project, including the late Hans Freudental, Andrejs Dunkels and Hilary Shuard, as well as Bruce Meserve and Marilyn Suydam, Alan Osborne and Margaret Kasten, Mogens Niss, Tibor Nemetz, Ubi D'Ambrosio, Brian Wilson, Tatsuro Miwa, Henry Pollack, Werner Blum, Roberto Baldino, Waclaw Zawadowski, and many others throughout the world.

Information about our project and future work can be found on the following webpages. Our Project Home Page: <http://math.unipa.it/~grim/21project.htm> leads directly to the paper proceedings of previous conferences from Egypt 1999 to Dresden 2009. [ProceedingsSouthAfrica](#) gives the proceedings of the South Africa conference in 2011. The proceedings of our last conference in Montenegro in 2014 are at <http://directorymathsed.net/montenegro/> For our Polish Superkurs Home Page and National Planning Meetings webpage see: www.cdnalma.poznan.pl (in Polish - but with pictures!) Andreas Filler at <http://www.afiller.de/charlotte07> has a photo album of our Charlotte Conference in 2007.

We are especially grateful this year to Professor Martin Stein of Münster University, the Owner and Manager of the company that will publish our printed proceedings: WTM-Verlag (Wissenschaftliche Texte und Medien – scientific texts and media) <http://www.wtm-verlag.de>

These Proceedings begin with the Plenary Paper by Douglas Butler (Autograph) followed by a list of titles and then the full text of the papers/workshops themselves, in alphabetical name order of the principal authors.

We sincerely thank all of the contributors for their time and creative effort. It is clear from the variety and quality of the papers that the conference has attracted many innovative mathematics educators from around the world.

I wish to thank especially Martin Stein, Jasia Morska and Douglas Butler for all their support and hard work in the preparation of these Proceedings.



Dr. Alan Rogerson

D.Phil (Oxon), M.Sc., B.Sc., B.A. (Lon), Dip.Ed., Cert. Ed. (Cantab).
Chairman of the International Program Committee
Co-ordinator of the Mathematics Education for the Future Project

Plenary Keynote Address: Technology must be transparent and not get in the way of teaching and learning

Douglas Butler, iCT Training Centre, Oundle, UK

Software and hardware solutions for mathematics teaching are evolving all the time, leaving many teachers bewildered by the ever increasing kaleidoscope of possibilities. Douglas will attempt to bring this audience up to date with some exciting lesson plans drawing on a new generation of hardware independent resources, the emphasis always being to let the mathematics shine through.

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Adenegan, Kehinde Emmanuel

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Galit Ashkenazi-Golan & Vered Gabai

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Clyde Felix

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**Evaluating Students' Mathematical Creative Thinking Involved in Modeling
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**We have no idea how capable children are: A multimodal analysis of children's
mathematical reasoning.**

Barbara Graves

**A Study of the Effect of using "What if Not" Strategy in Posing Geometry
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Majid Haghverdi & Maryam Gholami

**The Effect of Mathematics Reform Movements and Mathematical Discourse in
Pre-service Teachers' Ability to Design Problem-posing Situations**

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Bradford Hansen-Smith

Mathematical Habits of Mind: Fostering or Impeding

Gary A. Harris

Knowledge of Assessment and its implications

Hodaya (Liora) Hoch & Miriam Amit

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Elena Iurchenko

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Truth, Beauty and Goodness in Mathematics Education

Allan Tarp

An approach to assessing students' competences developed through math research

Ariana-Stanca Văcărețu

Colegiul National "Emil Racovita" Cluj-Napoca & Romanian RWCT Association
ariana.vacaretu@vimore.com

Abstract

This paper shares our approach to conceptualizing and assessing specific competences developed by students engaged in MATH.en.JEANS (MeJ) workshops. MeJ (Méthode d'Apprentissage des Théories Mathématiques en Jumelant des Etablissements pour une Approche Nouvelle du Savoir) workshops encourage students of different ages to engage in and eventually learn math by discovering and researching it.

Both the workshops and the pedagogical research and development were done within the Erasmus+ project *Learning math and languages through research and cooperation – MatLan*, supported by the European Commission. Based on findings from literature review and on the MeJ facilitator teachers' observations, we created a model to provide an overview of the competences developed through the MeJ workshops, defined and operationalised specific competences and prepared relevant assessment instruments. The paper shares samples of our proposed assessment instruments, which will be tested and validated within the next stage of our project.

Introduction

In the MeJ workshops held within the MatLan project, our students (aged 15 – 19) are placed in a math research context similar to professional research. Noticing that these workshops motivate students for learning and provide opportunities for developing various competences, we prepared assessment instruments to inform the outcomes of the workshops.

Competences developed within the MeJ workshops

To define the competences developed within this type of workshops, we first analysed the Researcher Development Framework (Vitae, 2010) and adapted the model to the context of the MeJ workshops. We included both transversal 21st-century skills and mathematical competences, as we consider that the MeJ workshops provide opportunities for both type of competences to develop. The diagram below shows the competencies potentially developed in the MeJ workshops.

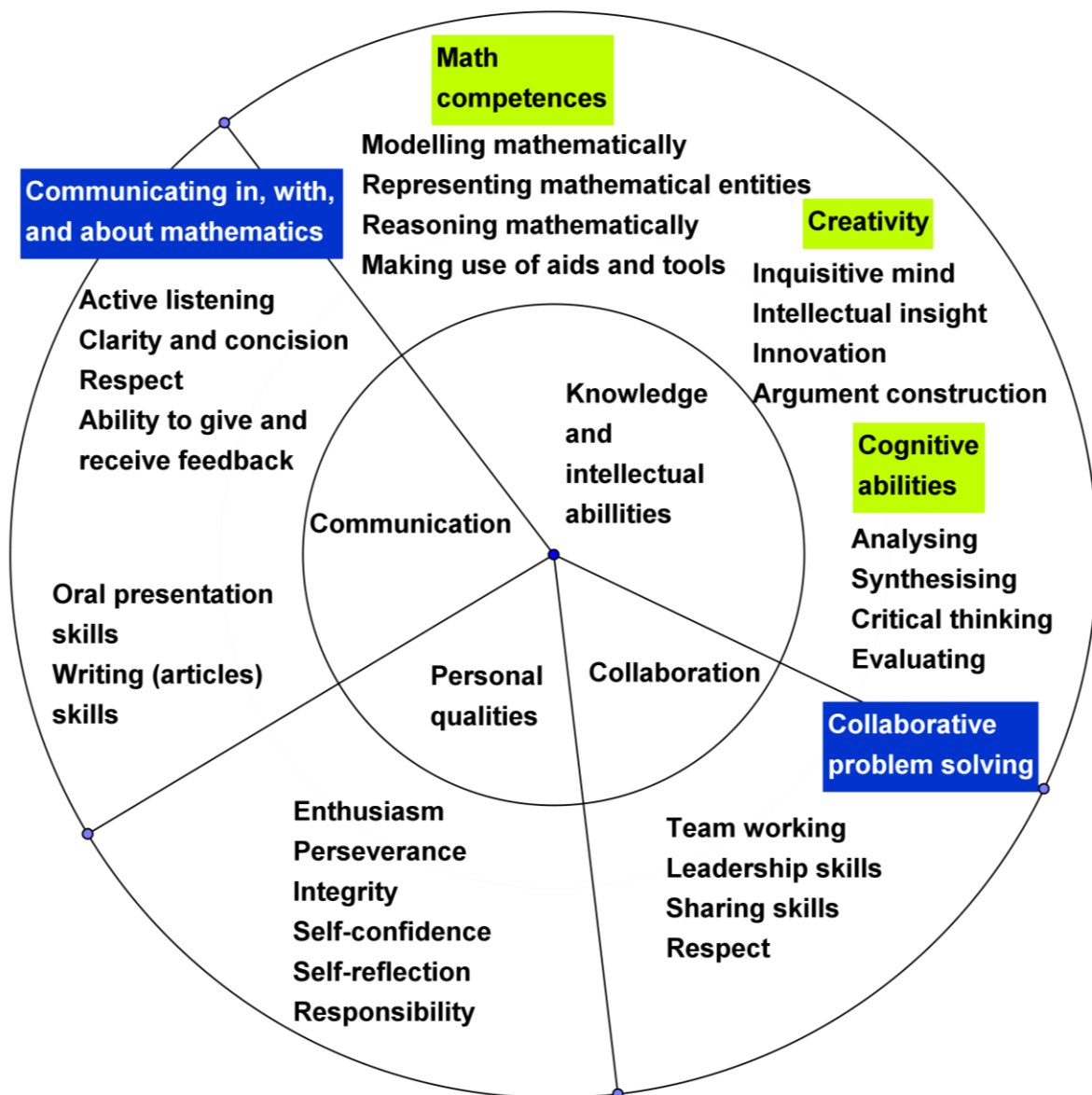


Figure 1: Competences developed within MeJ workshops

Our approach to developing the assessment instruments

To develop the assessment instruments, we started by defining each competence and making it visible. Observing what students do, make, say and write while researching the math topics in the MeJ workshops and analysing relevant literature on the respective competence, we identified the skills and sub-skills, as well as the student behaviour which demonstrates mastery of each identified sub-skill. Next, we described three levels of performance (novice, competent, expert) for each behavioural indicator and the developmental progression of the respective competence. Developmental progressions are tools which originally resulted from the work of Piaget (Hurst, n.d.). They are carefully designed detailed 'maps' which illustrate the increasingly sophisticated behaviours that a learner will display as they progress from being a novice to being an expert in any domain of learning

(University of Melbourne, 2015). Developmental progressions provide a tool for the teacher to be a systematic observer of student progress. They can help the teacher to notice the kinds of actions and behaviours that students demonstrate and to interpret these observations as evidence of skills and sub-skills growth. The developmental progression might be used as a frame of reference to interpret what they see the students doing, saying, making, or writing in MeJ workshops. Finally, we decided on the assessment methods and we designed the assessment tools by using the developmental progression for the respective competence.

An example: the collaborative problem solving (CPS) competence

A. Definition of the competence

Collaborative problem solving involves two or more people working towards a joint problem solution. All the participants must contribute their resources, skills, etc. so the problem can be solved. Those two or more people will have a common goal; the resources needed to solve the problem are beyond the capacity of either person alone. If they work together, they might be able to work it out. Collaborative problem solving, or working with others to solve a common challenge, includes the contribution and exchange of ideas, knowledge, or resources in order to achieve a shared goal.

This competence brings together two skills: problem solving - which can be included in the category Knowledge and intellectual abilities, related to ways of thinking (Blinkley, et al., 2012); and collaboration – which, in our model, is a category by itself, related to ways of working (Blinkley, et al., 2012).

In the MeJ workshop, mathematics research topics are launched by professional researchers. Small, 2-3-student groups, choose one of the proposed problems and do research to solve it. The students organize their work, identify the resources (strategies, knowledge, experience, equipment, software, materials); decide how the resources will be used for building and maintaining a shared understanding of the task and its solutions.

The major stages of the problem-solving process identified by Polya (1973) are appropriate for our research approach: understanding the problem, devising a plan, carrying out the plan, looking back (examining the solution obtained and checking the results). Both inductive and deductive reasoning are embedded within this process, as the research task challenges the students to detect information, identify patterns or analyse particular situations (as part of the inductive process), and then identify rules and test hypotheses (as part of the deductive process).

Within the research work, collaboration is essential – e.g. in our experience, when first confronted with the research topic, individuals had no idea from where to start. After discussing with their peers, ideas about how to start their research emerged. We engage the students in their zone of proximal development (Vygotsky, 1978). Students on the MeJ teams have different skills and knowledge, and each can be the ‘knowledgeable other’ in different moments of their work.

B. Making the competence visible

After observing the students researching the math topics in the MeJ workshop, and analysing the definition of collaborative problem solving skills (ATC21S Project, 2012) as well as Griffin's framework of cognitive and social skills (Griffin, 2014), we adapted Griffin's framework to the MeJ context.

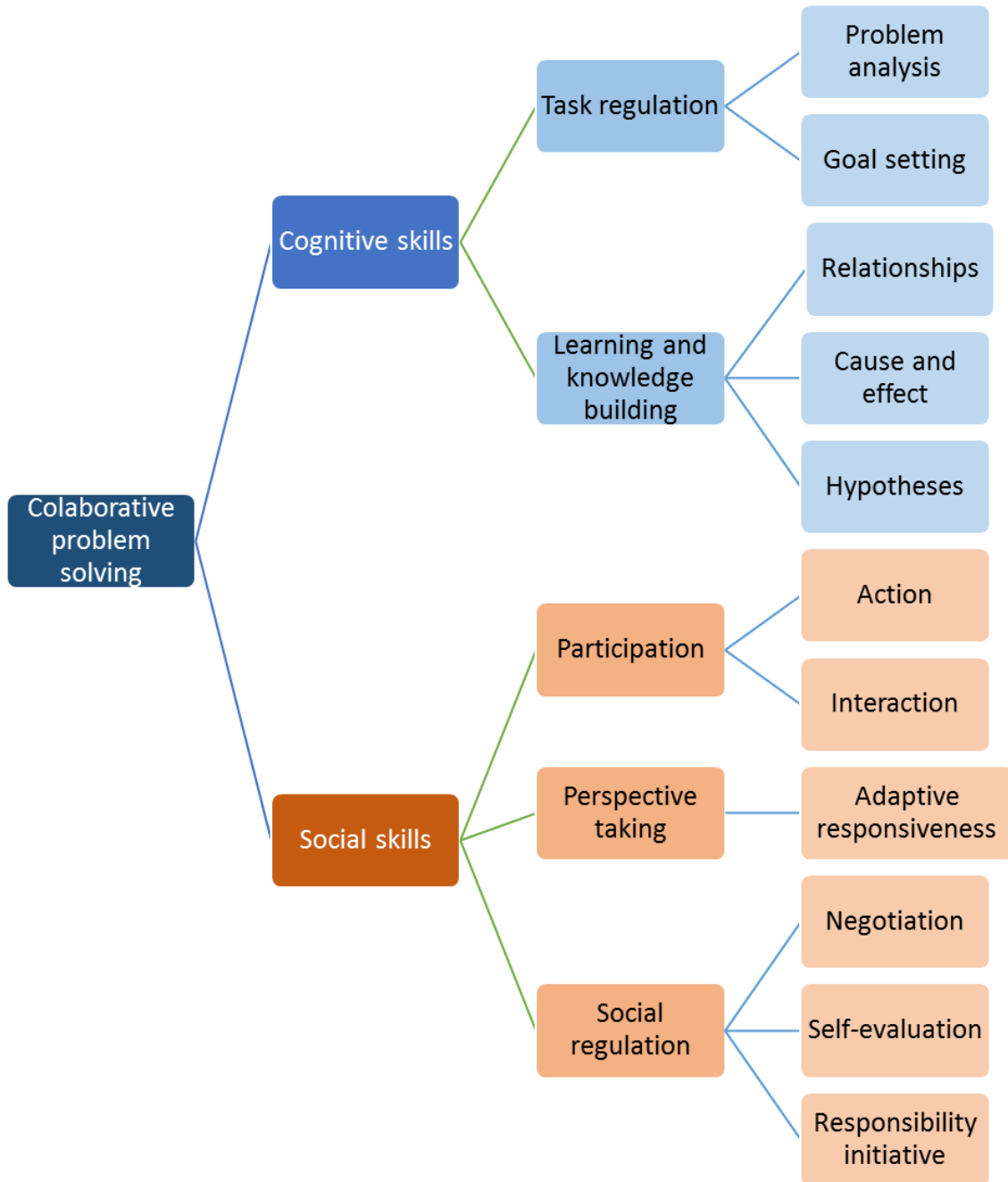


Figure 2: Skills and sub-skills of CPS

For each sub-skill, we identified the behaviour, the behaviour indicator and three performance levels. Below, we exemplify both, a cognitive and a social sub-skill.

Example 1. Cognitive sub-skill: Problem analysis

Behaviour: Student identifies the elements of the task/ problem space and information available for each element; they look for patterns and links between elements of the problem, and analyse particular cases.

Behaviour indicator: Analyses and describes the problem in familiar mathematical language.

Performance levels:

Low (novice): Describes the problem by reading it aloud. Explores 1 – 2 particular cases.

Medium (competent): Describes the problem by stating the given situation in his/ her own words. Divides the problem into subtasks. Explores 3 – 4 particular cases by using non-mathematical models (e.g. manipulatives).

High (expert): Describes the problem space in familiar mathematical language and states the hypotheses and the conclusion of the problem.

Sets/ defines the logical sequence of subtasks. Explores more than 4 particular cases by using ICT (e.g. GeoGebra) and/ or manipulatives.

Example 2. Social sub-skill: Adaptive responsiveness

Behaviour: Student integrates contributions of peers into his/ her own thoughts and actions.

Behavioural indicator: Ignoring, accepting or adapting contributions of others.

Performance levels:

Low (novice): Ignores contribution of peers.

Medium (competent): Responds to contributions of peers (e.g. considers contribution but doesn't make changes).

High (expert): Incorporates contributions of peers to suggest possible solution paths (e.g. makes changes based on contribution of peers).

C. The developmental progression

To describe the developmental progression of collaborative problem solving (CPS) within the MeJ workshops, we started from one dimension of the developmental progression empirically validated within the ATC21S project (ATC21S Project, 2014).

The developmental progression incorporates both social and cognitive skills.

Level	Level description
6 ↑	<p><i>The student works collaboratively through the problem solving process and assumes group responsibility for the success of the research tasks. The student works through the problem efficiently and systematically using only relevant resources. He/ she plans the research strategy based on a generalised understanding of cause and effect, and reorganizes understanding of the problem.</i></p> <p><i>The student tailors communication, incorporates contributions and feedback from peers and resolves conflicts.</i></p>
5	<p>The student identifies the necessary sequence of subtasks; explores more than four particular cases by using ICT (e.g. GeoGebra) and/ or manipulatives.</p> <p>The student's actions are planned and purposeful, identifying cause and effect and basing their goals on prior knowledge. He/ she reconstructs the understanding of the problem.</p> <p><i>The student promotes interaction and responds to peers' contribution but may not resolve differences.</i></p>
4	<p>The student divides the research task into subtasks, perseveres to successfully complete subtasks and simpler tasks, and explores four particular cases.</p> <p>The student identifies connections and/ or patterns in approaching the research task; he/ she identifies sequences of cause and effect, and modifies hypotheses.</p> <p><i>The student is aware of his/ her and the peers' abilities. They reach a common understanding, start to plan strategies for finding solutions and refine goals with their peers.</i></p>
3	<p>The student demonstrates effort towards finding solutions to the research task by stating the research topic in his/ her own words, and exploring three particular cases. <i>He/ she begins to share resources and information with the peers – resources and information shared are sometimes not relevant. The student sometimes asks for support from the teacher.</i></p> <p><i>The student reports his/ her activity.</i></p>
2	<p>The student attempts to better understand the problem through limited analysis; s/he explores two particular cases.</p> <p>He or she sets general goal, begins testing hypotheses.</p> <p><i>Interaction with peers is limited to brief acknowledgments on significant issues related to the research.</i></p>
1	<p>The student explores the problem space <i>independently with no evidence of collaboration.</i></p> <p>His/ her approach is unsystematic and focusing on isolated pieces of information; s/he explores one particular case.</p> <p><i>No evidence of participation, interaction with peers is limited to brief acknowledgments without providing information or resources.</i></p>

Table 1: Developmental progression of CPS within the MeJ context

D. Methods and instruments for assessing students' CPS

To assess students' CPS developed within the MeJ workshops, we plan to use the following methods:

- observation and analysis of entries in the students' logbooks (twice a year) using the CPS observation sheet - which is similar to the CPS developmental progression (see table 1);
- self-assessment (twice a year) using a CPS self-assessment sheet (see below).

Self-assessment sheet

Use the self-assessment sheet at the beginning of December and at the end of May of each school year. Mark with highlighters (different colours for the two dates) to identify the kinds of activities and behaviours you think you demonstrate.

Student Name:	Date 1: Date 2:	Explain your reasons for marking the statements. Give concrete examples of what you have done for supporting your choice.
<p>I work with others through the problem solving process and assume group responsibility for the success of our research tasks.</p> <p>I work together with my peers through the problem efficiently and systematically using only relevant resources. We plan(-ned) the research strategy based on a generalised understanding of cause and effect, and reorganize(-d) our shared understanding of the problem.</p> <p>I tailor communication with my peers, incorporate their contributions and feedback and resolve conflicts.</p>		
<p>I identify the necessary sequence of subtasks; I explore(-d) more than four particular cases by using ICT (e.g. GeoGebra) and/ or manipulatives.</p> <p>My actions are planned and purposeful, identifying cause and effect and basing my/ our goals on prior knowledge. I reconstruct(-ed) understanding of the problem.</p> <p>I promote interaction in our group and respond to my peers' contribution but may not always resolve differences.</p>		

Student Name:	Date 1: Date 2:	Explain your reasons for marking the statements. Give concrete examples of what you have done for supporting your choice.
<p>Together with my peers, we divided the research task into subtasks, and I worked hard to successfully complete subtasks/ simpler tasks. I explore(-d) four particular cases.</p> <p>I identify(-ied) connections and/ or patterns in approaching the research task; I identify(-ied) sequences of cause and effect, I modify(-ied) hypotheses.</p> <p>I am aware of my own and my peers' abilities. We reach(-ed) a common understanding of our research topic. We started to plan strategies for finding solutions and refined our goals.</p>		
<p>I put some effort into finding solutions to the research task by stating the research topic in my own words, exploring three particular cases. I started to share resources and information with my peers, but sometimes the shared resources and information were not relevant for our research. I sometimes asked for support from the teacher. I report(-ed) my activity on the research task to my group.</p>		
<p>I attempt(-ed) to better understand the research topic by reading the text of the problem aloud several times, I explore(-d) two particular cases.</p> <p>I'm aware that we have to complete the research task, I started testing hypotheses.</p> <p>In the interaction with my peers, I acknowledge communication on significant issues related to the research.</p>		
<p>I explore the problem space independently (work alone at home). My participation in the discussions during the weekly meetings is very little. I don't really understand the research topic. I focus on isolated pieces of information, I explore(-d) one particular case.</p> <p>I missed more than three weekly meetings, interaction with my peers is limited to brief acknowledgments without providing information or resources.</p>		

Table 2: CPS self-assessment sheet

The students' logbooks are useful only for identifying aspects mentioned in the observation sheet which the teacher could not observe during the MeJ meetings or online conferences or for verifying the observed actions and behaviour. If there is discrepancy between the teacher's observations and the student logbook, the teacher may decide to take a closer look at the specific discordant issues. The highest level of progression where all the activities or behaviour are marked will be considered the student's level in the development of the collaborative problem solving competence, while the level on the progression where about half the activities or behaviour are marked is the student's zone of proximal development.

In the logbook, the students write down what they have done to find solutions to the research topics during the weekly meetings and at home, tasks for the next meeting and their own reflections on what they have learned about themselves, about their peers and new (curricular area) knowledge acquired while doing the research.

The self-assessment sheet is discussed with the MeJ students at the beginning of the school year as they need to understand the statements; to this end, the teacher needs to model actions and behaviour and exemplify them.

An analysis of the significant differences between the results of the teacher's observations and student self-assessment is recommended.

Final thoughts

The assessment methods and instruments will be tested in the course of the next school year, and revisions will be operated as deemed necessary. We hope that our work will support teachers in assessing their students and identifying points of intervention where they are most likely to learn. In addition, we hope that these assessment instruments will help us provide evidence of the efficiency of MeJ workshops.

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